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The gauge fixing problem in the sp(3) BRST canonical formalism

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Abstract. The paper offers the complete form of the BRST charges and of the extended Hamiltonian for the first-rank theories in the context of the sp(3) BRST symmetry. A theorem is proposed in order to solve the problem of existence and uniqueness of a gauge fixed Hamiltonian in this context. As a general result, it is verified that the standard and the sp(2) theories could be recovered as particular cases of our generalized symmetry.

1. Introduction

The possibility of extension of the standard BRST symmetry towards a generalized one in the canonical quantization of gauge theories was suggested in [1], where the Koszul complex corresponding to the ghost momenta was constructed by setting the generators on more levels. Using the same pattern, it is possible to complete the theory and also to obtain the ghosts sector.

This paper develops the results reported in [2], where possible extensions of the standard BRST symmetry were presented, and where even a fourth-order symmetry was effectively built, but the gauge fixing problem was not discussed. In a previous paper [3] the existence of a gauge fixed sp(3) Hamiltonian was proved by using the three-graduation technique of the ghost fields, by generalizing the line of [4, 5]. Our objective is to prove the existence of a sp(3) gauge fixed Hamiltonian and to clarify the possible arbitrariness that could occur in its expression into the formalism proposed in [2]. The construction we propose generalizes the standard BRST theory [6], as well as the sp(2) BRST quantization procedure from [7, 8].

We shall focus now on the general case of the irreducible first-rank theories and we shall compute, for the first time, the full expressions of the three BRST charges and of the sp(3) BRST invariant Hamiltonian. The first-rank theories are a very important class of gauge theories, as long as the abelian gauge field, the Yang–Mills theories and other useful examples belong to it.

In order to obtain a quantum description of a gauge theory in the BRST context, the following algorithm must be applied:

- the extension of the phase space by adding new ghost-type variables to the initial coordinates; they are Grassmann-valued variables and assure the consistency of the theory;
- the construction of the anticommuting fermionic generating functions Ω_a, $a = 1, 2, \ldots,$ so as to satisfy some adequate boundary conditions; using the Poisson superbracket [7], extended to the whole phase space, the previous requirements are to be expressed by the relations:

$$[\Omega_a, \Omega_b] = 0 \qquad \epsilon(\Omega_a) = 1 \qquad a = 1, 2, \ldots \tag{1}$$

- the determination of the Hamiltonian that ought to be defined on the extended phase space and invariant under the global transformation of the variables given by the BRST charges Ω_a:

$$s_a H \equiv [H, \Omega_a] = 0 \qquad a = 1, 2, \ldots \tag{2}$$

- use of the gauge fixing procedure to obtain a properly defined Hamiltonian, free from the unphysical degrees of freedom.

The above-mentioned algorithm will be developed in this paper, which is structured as follows: in section 2 a short presentation of the extended phase space, suitable for the sp(3) Hamiltonian quantization of an irreducible gauge theory, will be considered; in section 3 the general method that can be used to obtain the BRST charges $\{\Omega_a, a = 1, 2, 3\}$ and the unitarizing Hamiltonian starting from equations (1) and (2) will be applied for the case of the first-rank theories. The results will be compared, in section 4, with the expressions of the same quantities as in the case of less extended symmetries. More precisely, we shall verify that the standard and the sp(2) BRST symmetries are particular cases of the sp(3) theory. Some concluding remarks will end the paper.

2. The sp(3) extended phase space

Let us consider a dynamical system described in the phase space of the canonical variables $\{q^i, p_i; i = 1, \ldots, n\}$ by the Hamiltonian $H_0(q, p)$ and by the set of irreducible first-class constraints $\{G_\alpha(q, p) = 0; \alpha = \overline{1, m}\}$. The following involution relations will be valid in this case:

$$[G_\alpha, G_\beta] = f^{\gamma}_{\alpha\beta}(q, p) G_\gamma \qquad [H_0, G_\alpha] = V^{\beta}_{\alpha}(q, p) G_\beta. \tag{3}$$

If the structure functions $f^{\gamma}_{\alpha\beta}$ and V^{β}_{α} do not depend on the canonical variables, that is to say they are constant coefficients, the system is called a first-rank system.

In the case when a *sp(3) symmetry* is wanted, the total BRST differential would be written as

$$s_T = s_1 + s_2 + s_3 \tag{4}$$

with the requirements

$$(s_T)^2 \equiv (s_1 + s_2 + s_3)^2 = 0 \tag{5}$$

$$s_a s_b + s_b s_a = 0 \qquad a, b = 1, 2, 3. \tag{6}$$

The *sp(3) symmetry* implies that, by construction, the triplet $\{s_a; a = 1, 2, 3\}$ will act on the generators of the phase space symmetrically with respect to the transformations of the symplectic group sp(3).

This larger symmetry that we impose on the system asks for a larger phase space. Let us denote by $M \equiv \{Q^A, P_A; A = 1, \ldots\}$ the new space, where Q^A and P_A designate all the generators and they are canonical conjugated with respect to a Poisson superbracket, which for two functionals F and G has the form [7]

$$[F, G] = \frac{\delta F}{\delta Q^A}\frac{\delta G}{\delta P_A} - (-1)^{\epsilon(F)\epsilon(G)}\frac{\delta G}{\delta Q^A}\frac{\delta F}{\delta P_A}. \tag{7}$$

2.1. The level structure and graduation rules

In order to obtain a consistent description of the theory, we shall construct an extended phase space which is obtained by adding some ghost variables to the real ones. We shall consider the stratification of the extended phase space on more 'levels' $L^{(l)}, l \in R$. Identical 'copies' of the same ghost-type variables could be present at various levels. As a result, the following four numbers are to be used in order to characterize the variables:

- the *ghost number (gh)* is defined in the standard manner: $gh > 0$ for ghosts and $gh < 0$ for ghost-momenta;
- the *resolution degree (res)* is defined only for the ghost momenta, so $res = -gh$;
- the *level number (lev)* is an integer (positive, negative or zero) characterizing the level on which the variables are situated.

As the resolution degree is connected to the ghost number, for a variable A we shall adopt the following representation:

$$A \equiv \overset{(g,l)}{A} \qquad g \equiv gh \qquad l \equiv lev. \tag{8}$$

In any 'composition' process of two variables A and B we suppose that the pair (g, l) would satisfy the superposition rule

$$\overset{(g_1,l_1)}{A} * \overset{(g_2,l_2)}{B} = \overset{(g_1+g_2,l_1+l_2)}{C}. \tag{9}$$

The constraints and all the observables of the theory are disposed on the $L^{(0)}$-level, which we shall call the *ground level*. Using the notation (8), we could write

$$G_\alpha \equiv \overset{(0,0)}{G_\alpha}.$$

The same representation will be used for the operators. The significance of the level degree will be connected in this case to the number of levels 'jumped' by the variables under the action of the operator. For the Koszul differentials, for example, we have [1]

$$\delta_c \equiv \overset{(-1,l)}{\delta_c} \qquad l(\delta_c) = c - 1 \qquad c = 1, 2, 3. \tag{10}$$

2.2. The ghost momenta spectrum

In the case of an irreducible gauge theory the structure of the Koszul complex suitable for the sp(3) quantization has been given in [1]. We shall briefly present here this structure and, starting from it, we shall obtain the whole extended phase space.

The Koszul complex will be generated by three sets of variables that will be situated on the 'ground' level and on other three levels:

$$\begin{aligned} P_A &\equiv \{\overset{(-1,l)}{P_{\alpha c}}; c = 1, 2, 3; l = c + 1\} \\ \pi_A &\equiv \{\overset{(-2,l)}{\pi_{\alpha c}}; c = 1, 2, 3; l = c - 4\} \\ \tau_A &\equiv \{\overset{(-3,l)}{\tau_\alpha}; l \equiv -3\}. \end{aligned} \tag{11}$$

On this complex we define a differential, δ_T, that could be decomposed into three anticommuting items:

$$\delta_T = \delta_1 + \delta_2 + \delta_3. \tag{12}$$

We shall consider the following action on the generators

$$\overset{(1,a-1)}{\delta_a}\ \overset{(-1,1-b)}{P_{\alpha b}} = \delta_{ab}\ \overset{(0,0)}{G_\alpha} \qquad a, b = 1, 2, 3$$
$$\overset{(1,a-1)}{\delta_a}\ \overset{(-2,b-4)}{\pi_{\alpha b}} = \epsilon_{abc}\ \overset{(-1,1-c)}{P_{\alpha c}} \qquad \epsilon_{123} = 1$$
$$\overset{(1,a-1)}{\delta_a}\ \overset{(-3,-3)}{\tau_\alpha} = \overset{(-2,a-4)}{\pi_{\alpha a}} \qquad a = 1, 2, 3 \tag{13}$$

where δ_{ab} represents the Kronecker symbol and ϵ_{abc} is the total antisymmetric tensor.

2.3. The canonical structure

In order to ensure the canonical structure of the phase space, to each momentum we add a coordinate conjugated to the momentum in respect to the bracket (7). We use the notation

$$Q^A = \{q^i, Q^{\alpha a}, \lambda^{\alpha a}, \eta^\alpha\}$$

where we suppose the following relations are true:

$$[Q^{\alpha a}, P_{\beta b}] = \delta^\alpha_\beta \delta^a_b \qquad [\lambda^{\alpha a}, \pi_{\beta b}] = \delta^\alpha_\beta \delta^a_b \qquad [\eta^\alpha, \tau_\beta] = \lambda^\alpha_\beta. \tag{14}$$

The properties of the ghost fields will be connected to those of the momenta:

$$\epsilon(G_\alpha) = \epsilon(\lambda^\alpha) = \epsilon(\pi_\alpha) \equiv \epsilon_\alpha$$
$$\epsilon(Q^\alpha) = \epsilon(P_\alpha) = \epsilon(\eta^\alpha) = \epsilon(\tau_\alpha) \equiv \epsilon_\alpha + 1 \tag{15}$$
$$g(Q^A) = -g(P_A) \qquad l(Q^A) = -l(P_A). \tag{16}$$

We are able now to present the whole structure of generators for the extended phase space, suitable for the sp(3) quantization of an irreducible gauge theory [1]:

$$M \equiv \{\tau_\alpha, \pi_{\alpha a}, P_{\alpha c}, p_i, q^i, Q^{\alpha a}, \lambda^{\alpha a}, \eta^\alpha\}. \tag{17}$$

3. The sp(3) construction for first-rank systems

3.1. The BRST charges

The sp(3) quantization of an irreducible system requires the determination of the BRST charges and of the extended Hamiltonian, which are solutions of equations (1) and (2), respectively, with adequate boundary conditions. Later we shall solve these problems and, to be more specific, we shall consider the particular case of a first-rank system, which is a system with constant structure functions $f^\alpha_{\beta\gamma}$ and V^α_β in relations (3).

In this section we obtain the exact form of the BRST charges by applying the homological perturbation theory. Let us consider the decomposition

$$\Omega_a = \sum_{j\geqslant 0} \overset{(j)}{\Omega}_a \qquad \mathrm{res}(\overset{(j)}{\Omega}_a) = j \qquad gh(\overset{(j)}{\Omega}_a) = 1. \tag{18}$$

The condition $s_a = \delta_a + \cdots$ induces the boundary expressions for the first items from (18):

$$\overset{(0)}{\Omega}_{ain} = G_\alpha \delta_{ab} Q^{\alpha b} \qquad \overset{(1)}{\Omega}_{ain} = \epsilon_{abc} P_{\alpha b} \lambda^{\alpha c} \qquad \overset{(2)}{\Omega}_{ain} = \pi_{aa} \eta^\alpha. \tag{19}$$

The Poisson superbracket (7) could also be decomposed in the form

$$[F, G] = \sum_{k=0}^{3} [F, G]_k \qquad \mathrm{res}([F, G]_k) = \mathrm{res}\, F + \mathrm{res}\, G - k. \tag{20}$$

We project equation (1) on different values of the resolution degree. For $k = 0$ we obtain

$$[\overset{(0)}{\Omega}_a, \overset{(0)}{\Omega}_b]_0 + [\overset{(1)}{\Omega}_a, \overset{(0)}{\Omega}_b]_1 + [\overset{(0)}{\Omega}_a, \overset{(1)}{\Omega}_b]_1 = 0.$$

As $\overset{(0)}{\Omega}_a \equiv \overset{(0)}{\Omega}_{ain}$, the solution of this equation is

$$\overset{(1)}{\Omega}_a \equiv \overset{(1)}{\Omega}_{ain} + \overset{(1)}{\Omega}_{a\sup} = \epsilon_{abc} P_{\alpha b} \lambda^{\alpha c} + \tfrac{1}{2} f^{\gamma}_{\alpha\beta} P_{\gamma m} Q^{\beta m} Q^{\alpha a}. \tag{21}$$

For $k = 1$, from (1) we obtain

$$[\overset{(1)}{\Omega}_a, \overset{(1)}{\Omega}_b]_1 + [\overset{(2)}{\Omega}_a, \overset{(1)}{\Omega}_b]_2 + [\overset{(1)}{\Omega}_a, \overset{(2)}{\Omega}_b]_2 = 0$$

and this allows us to obtain

$$\overset{(2)}{\Omega}_a = \pi_{aa} \eta^{\alpha} + \tfrac{1}{2} f^{\gamma}_{\alpha\beta} \pi_{\gamma m} \lambda^{\beta m} Q^{\alpha b} \delta_{ab} + \tfrac{1}{12} f^{\theta}_{\sigma\alpha} f^{\gamma}_{\theta\beta} \epsilon_{mnp} \pi_{\gamma m} Q^{\alpha n} Q^{\beta p} Q^{\sigma b} \delta_{ab}. \tag{22}$$

The last term appearing in (18) is $\overset{(3)}{\Omega}_a$. Its form might be given by projecting equation (1) for $k = 2$.

In conclusion, the BRST charges Ω_a, $a = 1, 2, 3$, which ensure the sp(3) BRST quantization of the first-rank theories have the form

$$\begin{aligned}\Omega_a \equiv \overset{(0)}{\Omega}_a + \overset{(1)}{\Omega}_a + \overset{(2)}{\Omega}_a + \overset{(3)}{\Omega}_a &= G_{\alpha} \delta_{ab} Q^{\alpha b} + \epsilon_{abc} P_{\alpha b} \lambda^{\alpha c} \\ &+ \tfrac{1}{2} f^{\gamma}_{\alpha\beta} P_{\gamma m} Q^{\beta m} Q^{\alpha a} + \pi_{aa} \eta^{\alpha} + \tfrac{1}{2} f^{\gamma}_{\alpha\beta} \pi_{\gamma m} \lambda^{\beta m} Q^{\alpha b} \delta_{ab} \\ &+ \tfrac{1}{12} f^{\theta}_{\sigma\alpha} f^{\gamma}_{\theta\beta} \epsilon_{mnp} \pi_{\gamma m} Q^{\alpha n} Q^{\beta p} Q^{\sigma b} \delta_{ab} + \tfrac{1}{2} f^{\gamma}_{\alpha\beta} \tau_{\gamma} Q^{\beta b} \eta^{\alpha} \delta_{ab} \\ &+ \tfrac{1}{12} [f^{\theta}_{\sigma\alpha} f^{\gamma}_{\theta\beta} + f^{\theta}_{\sigma\beta} f^{\gamma}_{\theta\alpha}] \tau_{\gamma} Q^{\sigma b} \delta_{ba} Q^{\beta n} \lambda^{\alpha n}.\end{aligned} \tag{23}$$

It is very important to note that if we were to use the assignment from table (17) and rule (9) in the previous expressions, we would find that the total BRST charge, defined as a sum of the three charges (23), is in fact split into three successive levels, $L^{(0)}$, $L^{(1)}$ and $L^{(2)}$,

$$\Omega^T = \overset{(10)}{\Omega}_1 + \overset{(11)}{\Omega}_2 + \overset{(12)}{\Omega}_3 . \tag{24}$$

3.2. The extended Hamiltonian

We now obtain an extension of the canonical Hamiltonian H_0 of a first-rank bosonic theory, as a solution of equations (2) with the boundary condition

$$H|_{G=P=\pi=\tau=0} = H_0. \tag{25}$$

We again use the homological perturbations theory, and we write

$$H = \sum_{r\geqslant 0} \overset{(r)}{H} \qquad \text{res}(\overset{(r)}{H}) = r \qquad gh(\overset{(r)}{H}) = 0. \tag{26}$$

It would be simple to verify that (2) projected for $r = -3, -2, -1$ would lead to identities. For $r = 0$ we obtain

$$[\overset{(0)}{H}, \overset{(0)}{\Omega}_a]_0 + [\overset{(1)}{H}, \overset{(0)}{\Omega}_a]_1 = 0.$$

As $\overset{(0)}{H} \equiv H_0(q, p)$ and using (19) for $\overset{(0)}{\Omega}$, the previous equation becomes

$$\overset{(1)}{H} = V^{\alpha}_{\beta} P_{\alpha a} Q^{\beta a}. \tag{27}$$

The equation for $r = 1$ has the form

$$[\overset{(1)}{H}, \overset{(1)}{\Omega}_a]_1 + [\overset{(2)}{H}, \overset{(0)}{\Omega}_a]_1 + [\overset{(2)}{H}, \overset{(1)}{\Omega}_a]_2 = 0.$$

This equation gives the term

$$\overset{(2)}{H} = V^{\alpha}_{\beta}\pi_{\alpha a}\lambda^{\beta a}. \tag{28}$$

The last non-vanishing term in the decomposition (26) will correspond to the value $r = 2$ and will have

$$\overset{(3)}{H} = V^{\alpha}_{\beta}\tau_{\alpha}\eta^{\beta}. \tag{29}$$

In conclusion using the graduation from (17), for a first-rank irreducible gauge theory we can extend the Hamiltonian $H_0(q, p)$ to the whole phase space in the form

$$\begin{aligned} H = H_0 + V^{\alpha}_{\beta} \overset{(-1,0)}{P_{\alpha}} \overset{(1,0)}{Q^{\beta}} + V^{\alpha}_{\beta}[\overset{(-1,-1)}{P_{\alpha}} \overset{(1,1)}{Q^{\beta}} + \overset{(-2,-1)}{\pi_{\alpha}} \overset{(2,1)}{\lambda^{\beta}}] \\ + V^{\alpha}_{\beta}[\overset{(-1,-2)}{P_{\alpha}} \overset{(1,2)}{Q^{\beta}} + \overset{(-2,-2)}{\pi_{\alpha}} \overset{(2,2)}{\lambda^{\beta}}] + V^{\alpha}_{\beta}[\overset{(-2,-3)}{\pi_{\alpha}} \overset{(2,3)}{\lambda^{\beta}} + \overset{(-3,-3)}{\tau_{\alpha}} \overset{(3,3)}{\eta^{\beta}}]. \end{aligned} \tag{30}$$

In this expression we can see that the first two terms contain variables from the $L^{(0)}$-level. The third term is a $L^{(-1)} \otimes L^{(1)}$ term and the last two brackets are expressed as $L^{(-2)} \otimes L^{(2)}$ and $L^{(-3)} \otimes L^{(3)}$ terms, respectively.

3.3. The gauge fixing

We come now to the analysis of the gauge fixing procedure that should be used in the sp(3) case.

In the standard BRST theory the gauge fixed Hamiltonian has the form

$$H_{gf} = H + [\Omega, Y].$$

The only requirements on Y consist of using manifestly covariant gauges and $gh(Y) = -1$. There are many possibilities for enlarging the extended phase space without changing the physical contents of the theory, but only the simpler ones have been used in the practical cases [9].

For the sp(2) case the gauge fixing is accomplished so that

$$H_{gf} = H + [\Omega_1[\Omega_2, Y]].$$

In the formalism we have adopted, the sp(3) BRST invariant Hamiltonian will have the form

$$H_{gf} = H + s_T Y \equiv H + K. \tag{31}$$

The unique standard requirement is $gh(H_{gf}) = 0$ and it simply implies $gh(K) = 0$. No restriction on the values of the level number must be imposed. Therefore, we could consider that

$$K = \sum_{l=-\infty}^{+\infty} \overset{(0,l)}{K}. \tag{32}$$

In the standard BRST theory variables of the form $\overset{(0,0)}{A} \in L^{(0)}$ appear, so that the sum (32) contains the term $\overset{(0,0)}{K}$ only. For the sp(3) theory one can find $K = s_T Y$ of the form (32) with the properties:

(i) K is s_a-invariant ($a = 1, 2, 3$);
(ii) for any $a = 1, 2, 3$, there is a function Y' s.t. $K \equiv s_T Y = s_a Y'$.

These statements are connected to the uniqueness of the sp(3) Hamiltonian and the form of the arbitrary terms that the sp(3) invariance allows to appear in the extended Hamiltonian. More precisely, it is simple to verify the following theorem, which generalizes a result issued from the sp(2) case [7], would be true.

Theorem 1. *Let K be a sp(3)-BRST invariant non-constant function. A function Y exists, defined on $M \equiv \{Q^A, P_A; A = 1, \ldots\}$ with $gh(Y) = -3$ so that*

$$K = \frac{1}{3!}\epsilon^{abc}[\Omega_a, [\Omega_b, [\Omega_c, Y]]]. \tag{33}$$

In conclusion, as we require the sp(3) invariance of H_{gf} and H from (31) to satisfy this property by construction, we obtain that K should have the form (33).

4. Comparison with previous BRST symmetries

The standard BRST quantization for an irreducible theory could be performed in the extended phase space:

$$\begin{aligned} &M_0 = \{P_\alpha, p_i, q^i, Q^\alpha\} \\ &gh P_\alpha = -gh Q^\alpha = -1 \qquad \text{res } P_\alpha = 1 \qquad \text{res } Q^\alpha = 0. \end{aligned} \tag{34}$$

The BRST charge for a bosonic first-rank theory has the form, following [9],

$$\Omega = G_\alpha Q^\alpha + \tfrac{1}{2} f^\gamma_{\alpha\beta} P_\gamma Q^\beta Q^\alpha. \tag{35}$$

The extended Hamiltonian can be written as

$$H = H_0 + V^\alpha_\beta P_\alpha Q^\beta. \tag{36}$$

In the case of the sp(2) symmetry, the suitable phase space ought to be generated by the variables

$$M_1 \equiv \{\pi_\alpha, P_{\alpha a}, p_i, q^i, Q^{\alpha a}, \lambda^\alpha\}. \tag{37}$$

For the two BRST charges corresponding to a first-rank theory we obtain [10]

$$\begin{aligned} \Omega_a = {} & G_\alpha Q^{\alpha b}\delta_{ab} + \epsilon_{ab} P_{\alpha b}\lambda^\alpha + \tfrac{1}{2} f^\gamma_{\alpha\beta} P_{\gamma m} Q^{\beta m} Q^{\alpha b}\delta_{ab} \\ & + \tfrac{1}{2} f^\gamma_{\alpha\beta}\pi_\gamma\lambda^\beta Q^{\alpha b}\delta_{ab} + \tfrac{1}{12} f^\theta_{\sigma\alpha} f^\gamma_{\theta\beta}\epsilon_{mn}\pi_\gamma Q^{\alpha m} Q^{\beta n} Q^{\sigma b}\delta_{ab}. \end{aligned} \tag{38}$$

The total ‘supersymmetric’ Hamiltonian has the form

$$H = H_0 + V^\alpha_\beta P_{\alpha a} Q^{\beta a} + V^\alpha_\beta \pi_\alpha \lambda^\beta. \tag{39}$$

We can now compare these results to those we obtained in the previous section.

- We can assign a *level number* to the variables from (34), so that

$$P_\alpha = \overset{(-1,0)}{P_\alpha} \qquad Q^\alpha = \overset{(1,0)}{Q^\alpha}.$$

It is clear that the BRST charge (35) and the Hamiltonian (36) of the standard theory are created using the $L^{(0)}$ variables only. They might be considered as the degree zero approximation of the similar quantities (23) and (30).

- In the case of the variables (37) we can assign the *level number* so that

$$P_{\alpha a} = \overset{(-1,1-a)}{P_\alpha} \qquad Q^{\alpha a} = \overset{(1,a-1)}{Q^\alpha} \qquad \pi_\alpha = \overset{(-2,-1)}{\pi_\alpha} \qquad \lambda^\alpha = \overset{(2,1)}{\lambda^\alpha} \qquad a = 1, 2.$$

In our notation, the variables π_α and λ^α have been identified as the $L^{(-1)}$ and $L^{(1)}$ variables, respectively. With these assignments, the charges (38) have the form

$$\Omega_1 = \overset{(1,0)}{\Omega} \qquad \Omega_2 = \overset{(1,1)}{\Omega}.$$

The Hamiltonian (39) is the sum of H_0 and two terms built with the $L^{(0)}$ and $L^{(-1)} \otimes L^{(1)}$ variables, respectively. So, we could identify the sp(2) theory as a second-order theory (a 'two-level' theory).

5. Conclusions

We had in mind two main aims for the present paper. First, we wished to offer an explicit example of computation of an sp(3) BRST symmetry in order to compare this symmetry with those previously developed, which were less extended symmetries. We chose the example of first-rank theories because of their practical importance and because they allow us to obtain the complete expressions for the BRST charges and for the Hamiltonian. Second, we tackled the gauge fixing problem and we have presented here the structure of a sp(3) gauge fixed Hamiltonian. The conclusions we infer are the following.

(i) It is possible to extend the phase space more than the sp(2) theory shows. An extended phase space allows the implementation of an extended BRST symmetry. The theory we proposed offers a rule for writing all the extra terms that could be expected for a given structure of the phase space.

(ii) In the level picture that we propose, the BRST charges and the extended Hamiltonian could be viewed as sums of more items, situated on successive levels. From this point of view, the standard BRST theory is a one-level theory and the sp(2) is a two-level theory. We can restrict ourselves to the number of levels we want for the case we are dealing with.

(iii) the freedom of choosing a gauge fixing term is as large as the symmetry implemented. In the level picture, this extended freedom is given by the possibility of using a 'more levels' function, of the form (32).

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